Haoming Wang

Probabilistic Methods in Combinatorics

A Comprehensive Guide to Multivariate Statistics

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Preprint Version

Probabilitas non servitur ad creandum, sed ad intellegendum. – Probability is not served for creation, it is for understanding.

Foreword

Abstract: This book introduces several statistical distributions, like normal distributions, gamma distributions, beta distributions, etc on fibres of Grassmannians, with the exact distribution of the characteristic function and characteristic roots given by the symmetric function. We study the ampleness of such statistical distributions on fibres and classify complete double flags via these results.

Keywords: Integer partition · Young tableaux · Grassmannian · Distribution on fibres

Tientsin, January 2026 *Haoming Wang*

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TBA

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[LB15, LB18]

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